Day 10

Inverse Kinematics

Inverse Kinematics

- given the pose of the end effector, find the joint variables that produce the end effector pose
- for a 6-joint robot, given

$$T_6^0 = \begin{bmatrix} R_6^0 & o_6^0 \\ \mathbf{0} & 1 \end{bmatrix}$$

find

$$q_1, q_2, q_3, q_4, q_5, q_6$$

RPP + Spherical Wrist

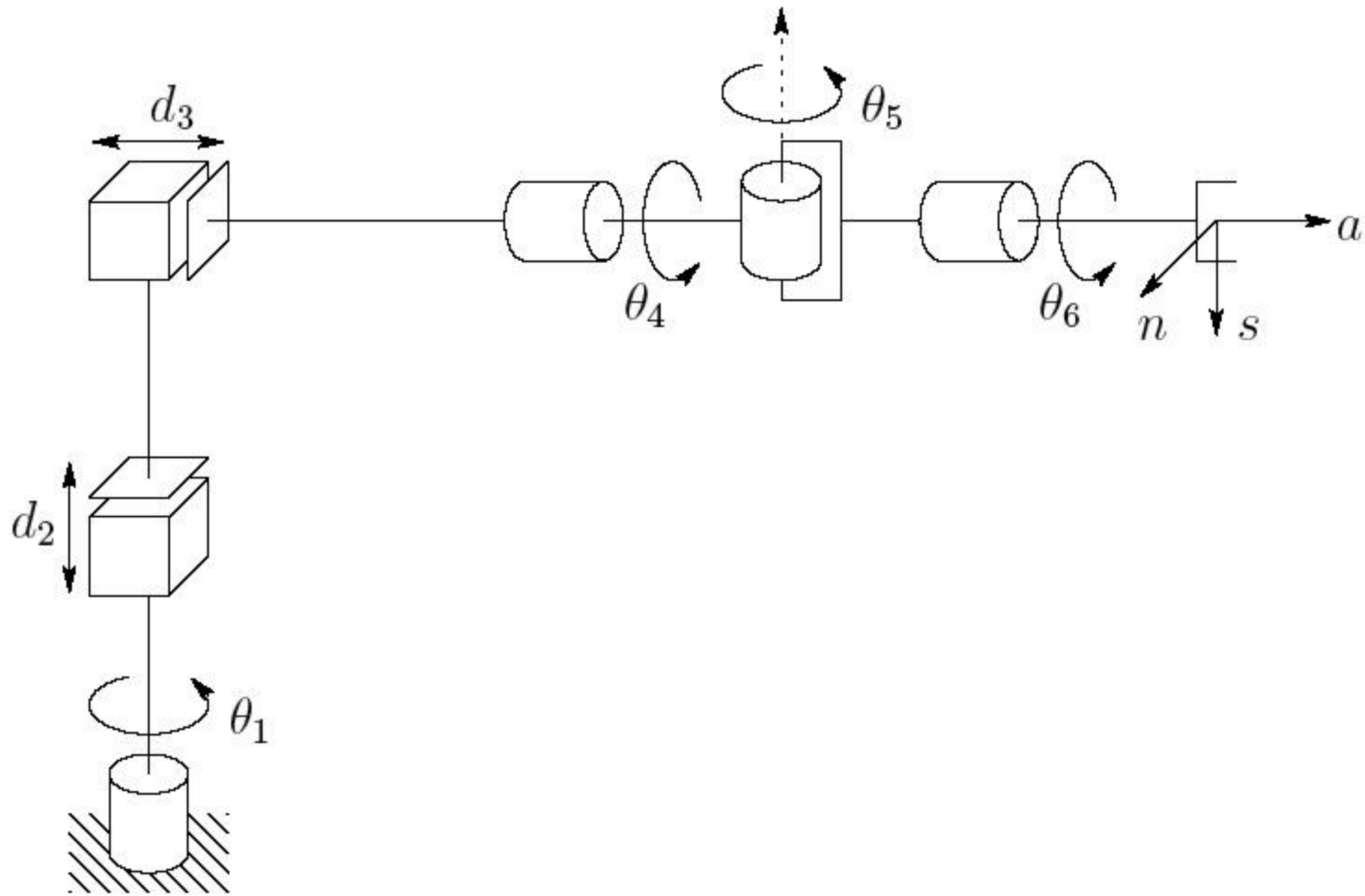


Figure 3.9: Cylindrical robot with spherical wrist.

RPP + Spherical Wrist

- solving for the joint variables directly is hard

$$T_6^0 = T_3^0 T_6^3 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$r_{11} = c_1 c_4 c_5 c_6 - c_1 s_4 s_6 + s_1 s_5 c_6$$

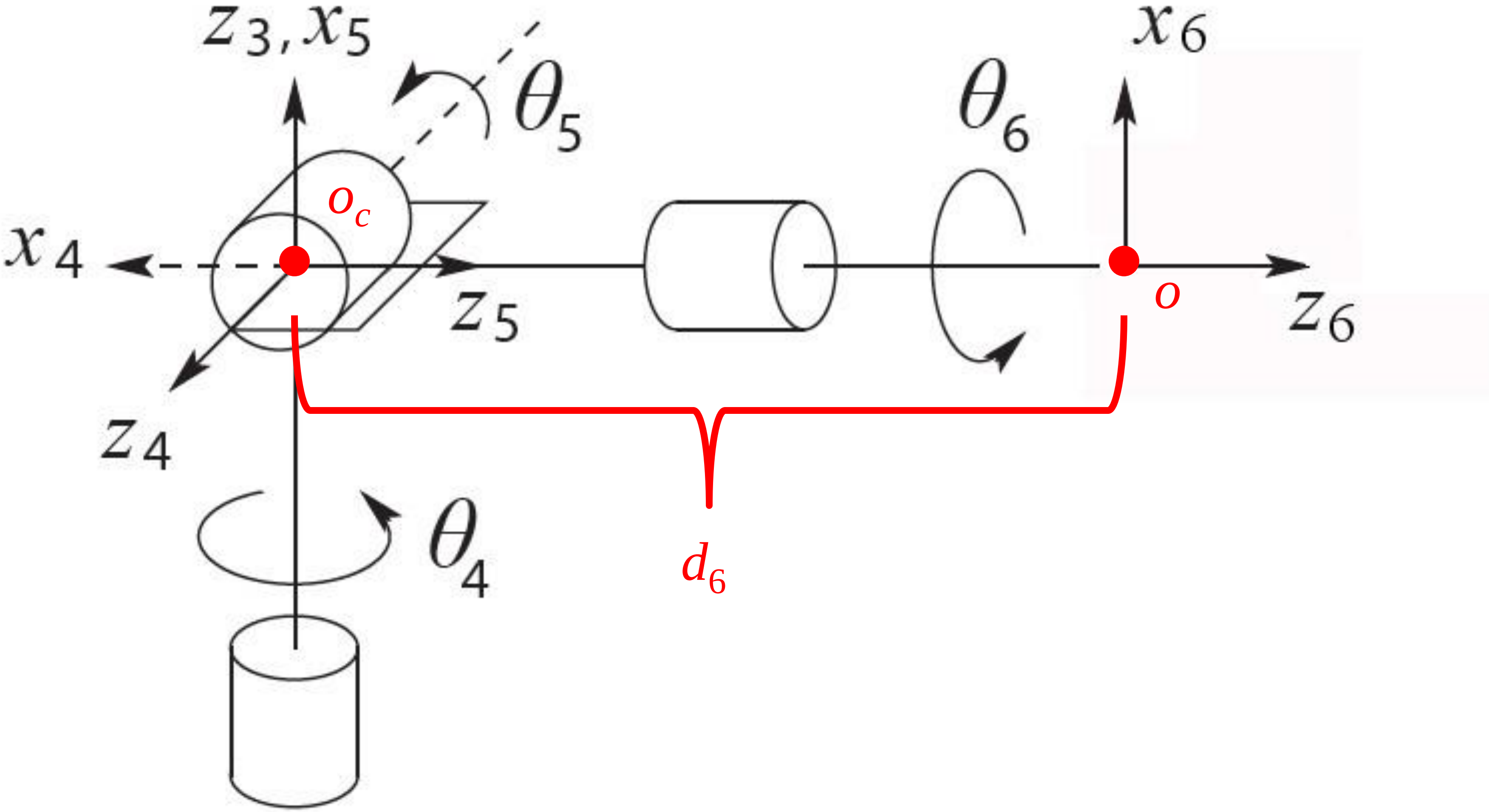
$$\vdots$$

$$d_z = -s_4 s_5 d_6 + d_1 + d_2$$

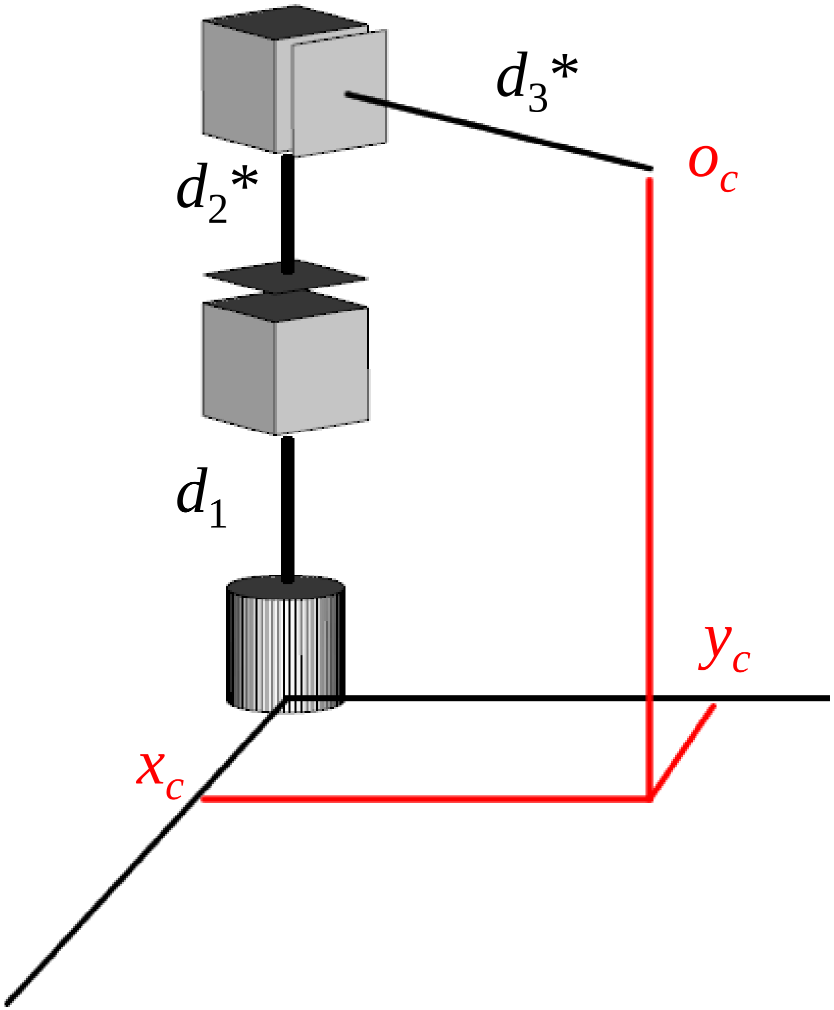
Kinematic Decoupling

- for 6-joint robots where the last 3 joints intersecting at a point (e.g., last 3 joints are spherical wrist) there is a simpler way to solve the inverse kinematics problem
  1. use the intersection point (wrist center) to solve for the first 3 joint variables
     - inverse position kinematics
  2. use the end-effector pose to solve for the last 3 joint variables
     - inverse orientation kinematics

Spherical Wrist



RPP Cylindrical Manipulator



RRP Spherical Manipulator

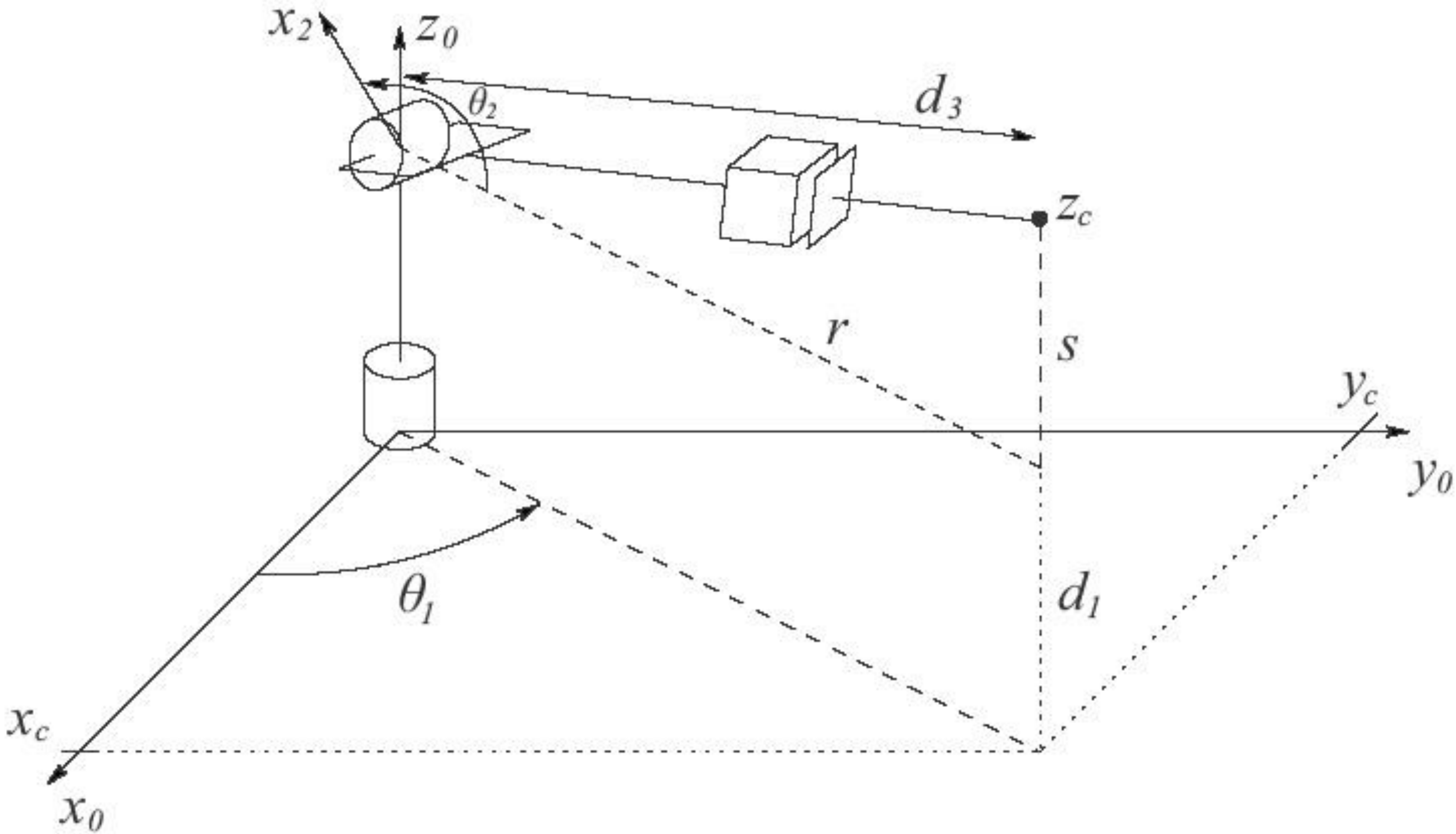
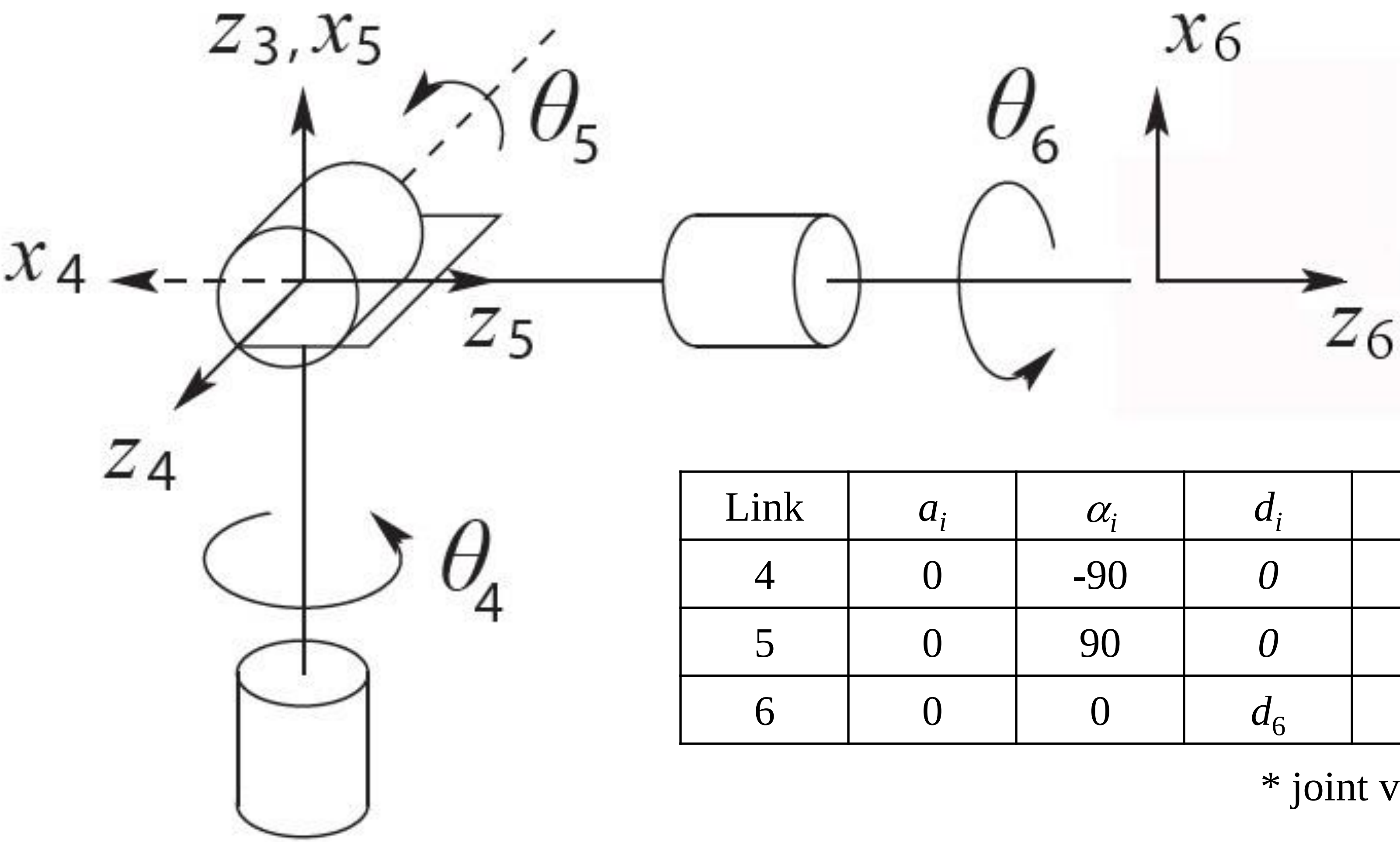


Figure 3.21: Spherical manipulator.

Spherical Wrist



Link	a_i	α_i	d_i	θ_i
4	0	-90	*0*	θ_4*
5	0	90	*0*	θ_5*
6	0	0	d_6	θ_6*

* joint variable

Spherical Wrist

$$T^3_6 = T^3_4 T^4_5 T^5_6 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$